Determining Characteristics of a RRC Circuit

Riley Kenyon

February 5th, 2019

Abstract

This report describes the analysis of an RRC circuit as a signal and system, both theoretically and experimentally. The characteristics observed were recorded with a step response and a frequency response function.

1 Introduction

The RRC circuit is an easy to measure, small, cheap, and overall great representation of a first order system. Figure 1 shows the schematic of the circuit and the terminals where the voltage is supplied and measured. System identification is performed by analyzing the step response of the circuit, where the value of σ (the only pole location) can be found experimentally.

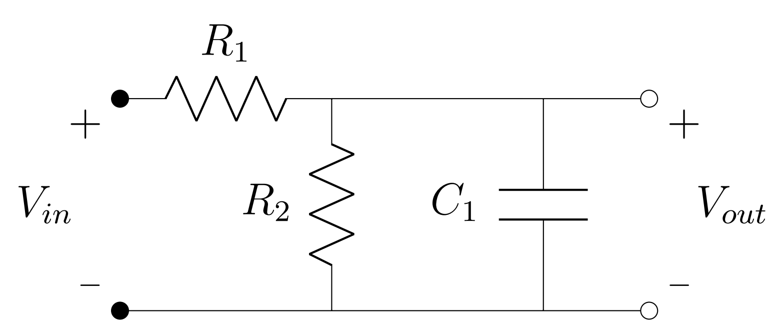

Figure 1: RRC Circuit

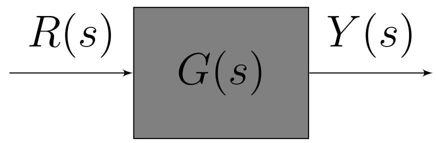

Figure 2: Open Loop Block Diagram

Performance characteristics such as rise time aid in the confirmation of component values, and ultimately results in a better fitting model of the system by comparing σ to a theoretically derived value. Where the value of σ is influential in determining both performance and frequency response. When analyzing the the system, the circuit should be thought of as a system and a signal. Here, a signal is a description of how one parameter varies with another. Monitoring how output voltage changes with respect to an input voltage step is an example of a signal. A system is any process that produces an output signal in response to an input, such as a varying the frequency or amplitude of the input to a circuit. A signal can be manifested as a function with respect to time. The voltage output of the circuit can be represented by a function when subjected to a voltage step. A system can be represented as a block diagram as in 2 and with a frequency response function (or Bode Plot). As previously stated, the results from both can aid in system identification of component parameters and will be the objective of the lab. Due to the system being in open-loop control, there is no feedback loop or additional controller. As represented in Figure 2 there is a single transfer function relating output voltage and input voltage.

2 Theoretical Transfer Function

The governing equation of state for an RRC circuit is shown in Equation 1, which can be derived by breaking up the system into idealized circuit elements.

$$R_1C_1\dot{V}_{out} + \frac{R_1 + R_2}{R_2}V_{out} = V_{in}, \tag{1}$$

By taking the Laplace Transform of the differential equation, the resulting transfer function of the system is represented by

$$G(s) = \frac{V_{out}}{V_{in}} = \frac{R_2}{R_1R_2C_1s + R_2 + R_1}, \tag{2}$$

where for the purposes of this lab, the values of R_1 and R_2 are assumed to be $5k\Omega$, and the capacitor C_1 to be 2.2 µF.In the initial analysis of this system, by assuming $R_1 = R_2$ the preliminary transfer function $G(s)$ can be simplified to

$$G(s) = \frac{V_{out}}{V_{in}} = \frac{1}{RCs + 2} \tag{3}$$

As a reminder, a first order can also be represented in standard form by Equation 4

$$G(s) = \hat{k}\frac{\sigma}{s + \sigma}, \tag{4}$$

where σ is $\frac{2}{RC}$ and $\hat{k}$ is $\frac{1}{2}$. This is useful in for creating a preliminary model of the system and estimating performance.

At this point, we can calculate a theoretical rise time for the system, as well as steady state error. For the assumed resistor and capacitor values, the rise time is $t_r = 12.1$ ms calculated form the relation in Equation 5 for a first order system. This estimate corresponds to a theoretical $\sigma_{theo} = 181.82$.

$$t_r = \frac{2.2}{\sigma} \tag{5}$$

Experimental data will be cross-referenced to confirm the validity of the model, described in section 2 where an adapted model of the system is formed based upon the step response of the system and the physical values of the resistors in the circuit. This information can be used to estimate the actual value of the capacitor and gain a more accurate model.

3 Obtaining Measurements

The experimental results are obtained via the methods below. To understand the RRC circuit as a signal, the step response is recorded along with rise time (settling time was not deemed necessary to measure because the RRC is a first order system and σ can be calculated from rise time alone). As a system, the frequency response function will be formed based on some theory and baseline measurements of attenuation and phase shifting.

3.1 Procedure

3.1.1 Step Response

1. From the function generator produce a 1 Vpp, 500mv offset, 1 Hz square wave to act as a 1 V step. Connect to the input of the circuit, Found in Figure 1.

2. Tee off the waveform generator and connect another BNC cable to channel 1 of the oscilloscope, ensure that the acquisition of the waveform is using the "High-res" smoothing.

3. Using an oscilloscope probe, record the output V_{out} in channel 2 of the oscilloscope. (Don't forget to change the amplification to 10:1 if it is not set).

4. Save the oscilloscope .png file and the .csv with 10,000 samples.

3.1.2 Frequency Response Data

1. From the function generator produce a 1 Vpp, 500mv offset Sine wave at 10 Hz.

2. Tee off the waveform generator and connect another BNC cable to channel 1 of the oscilloscope.

3. Using an oscilloscope probe, record the output V_{out} in channel 2 of the oscilloscope.

4. Add measurements of phase and amplitude to the oscilloscope for both channels.

5. Save the oscilloscope .png file and the .csv with 10,000 samples.

6. Repeat steps 1 - 5 with frequencies of [10 30 100 300 3000] Hz.

3.2 Experimental Results

From the data collected, an experimental rise time was measured to be $t_r =$ 12.233 ms and have a step response shown in Figure 3.

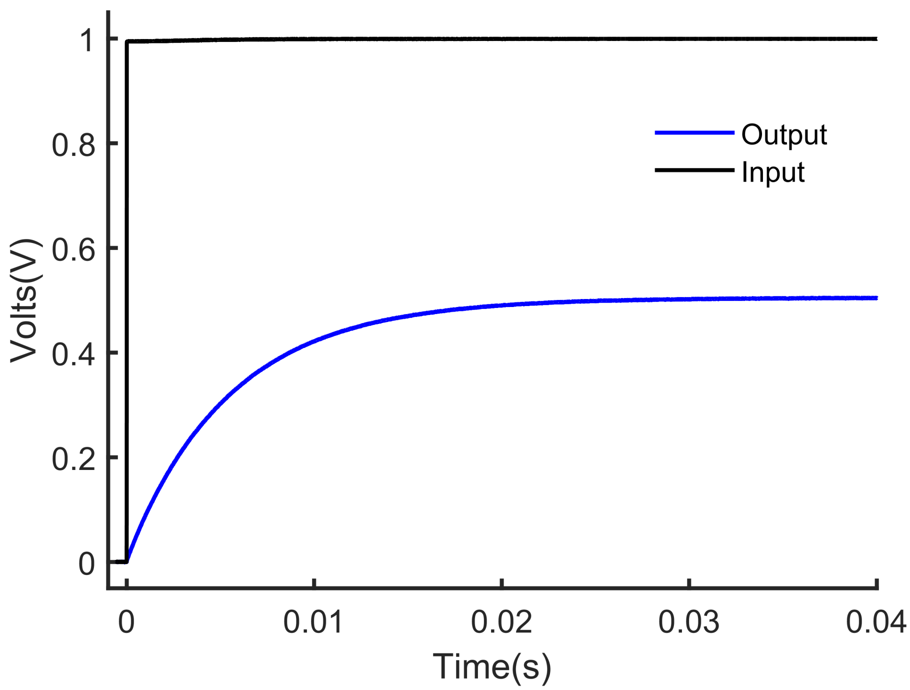

Figure 3: The RRC step response

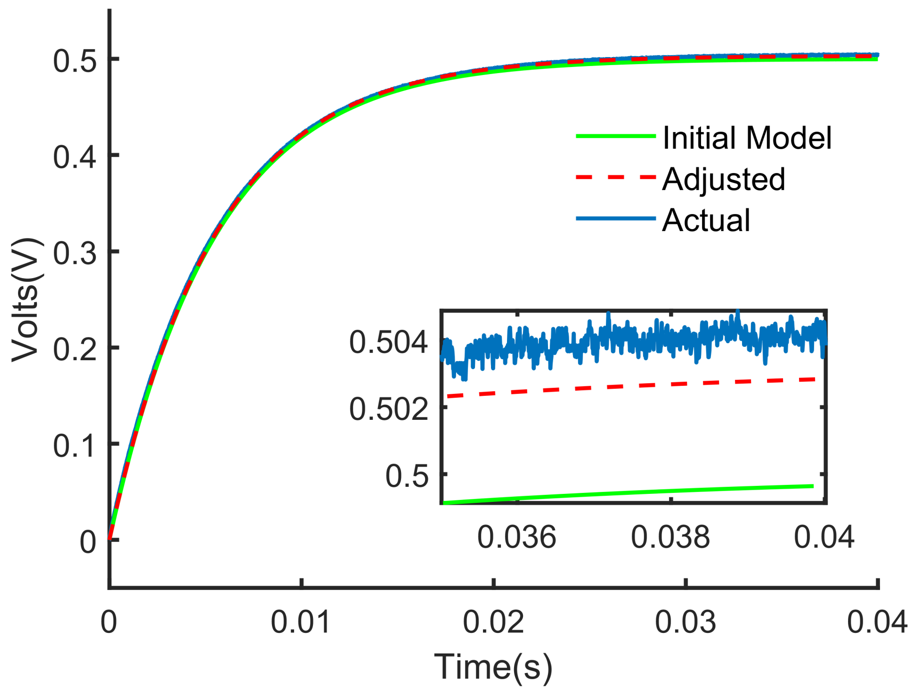

Figure 4: The RRC step response

Using Equation 5, the experimental value of $\sigma_{exp} = 179.84$ which is within 2% of the theoretical $\sigma_{theo} = 181.82$. To adjust the model with system identification, σ can be used to solve for the actual capacitor value. Measuring the resistors give values of $R_1 = 4.976k\Omega$, and $R_2 = 5.040k\Omega$. Solving Equation 5 with the experimental value of σ and the measured resistances yield a capacitor value of $C_1 = 2.22\mu F$ which is within the tolerance of the capacitor (20%). The adjusted theoretical step response can found in Figure 4 along with the initial theoretical model and the experimental response. Looking at the circuit as a system, the value of σ_{exp} corresponds to a cutoff frequency of 179.84 Rad/s which is equal to 28.6 Hz. To test the system's response to a varying sine-wave frequencies. The circuit was sampled at a variety of frequencies to obtain a better understanding of the system as a filter. In Figure 5 visualizes the frequency response of the RRC with a theoretical bode plot overlaid with experimental data.

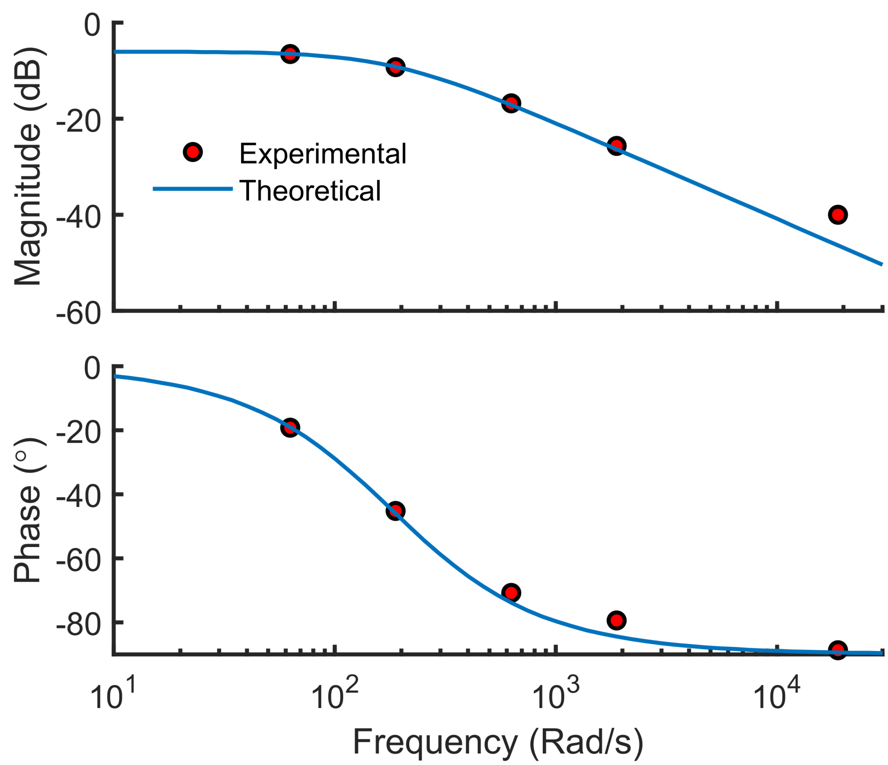

Figure 5: The Bode plot of the system with magnitude and phase plots

4 Conclusion

The RRC circuit is a first order system, with an experimentally determined pole of 179.84. In order to improve the system and decrease the steady-state error you can implement a proportional gain feedback loop into the circuit and shift the pole further left to decrease rise-time and adjust the steady-state value. The circuit acts as a low pass filter with a cutoff frequency of 28.6 Hz, visualized in Figure 5. The performance characteristics of the system can be quantified with a rise time of $t_r = 12.233$ ms and a DC-gain of 0.504, both of which were experimentally determined. To create a better representation of the system, the model was updated to reflect the physical values of the resistors and then fitted to a capacitor value with the experimental step response (as seen in Figure 4).

A Code

A.1 Creating Plots and Measuring Performance

```
%% Lab 01 - Industrial Automation - Riley Kenyon
close all; clear all; clc;
%% Experimental
data0 = csvread('step.csv',2,0);
y3 = data0(:,3)-data0(1,3); % change to zero offset
stepInput = data0(:,2)-data0(1,2);% change to zero offset
t3 = data0(:,1);
figure()
hold on
plot(t3,y3,'LineWidth',1.5);
xlabel('Time(s)')
ylabel('Volts(V)')
axis([0,0.04,-0.05,0.55]);
%rt(1) = risetime(data0(:,2),data0(:,1));
%% Theoretical
R(1) = 4976; %5070;
R(2) = 5040; %5019;
C = 2.205e-6;
sys1 = tf(R(2),[R(1)*R(2)*C, (R(1) + R(2))]);
```

```
sys = tf(1,[5000*C, 2]);
[y1,t1] = step(sys,'g');
[y2,t2] = step(sys1,'r--');
plot(t1,y1,'g','LineWidth',1.5)
plot(t2,y2,'r--','LineWidth',1.5);
%% Plotting
axes('position',[.45 .175 .4 .3])
box on % put box around new pair of axes
index = 0.035<t1 & t1<0.04;
hold on
plot(t1(index),y1(index),'g','LineWidth',1.5);
plot(t2(index),y2(index),'r--','LineWidth',1.5)
plot(t3(7732:8819),y3(7732:8819),'LineWidth',1.5);
axis tight
legend('Initial Model','Adjusted','Actual');
%% Step Response Plot
figure()
hold on
plot(t3,y3,'b','LineWidth',1.5);
plot(t3,stepInput,'k','LineWidth',1.5);
axis([-0.001 0.04 -0.05 1.05]);

% Frequency data
f = (2*pi*[10 30 100 300 3000])';
mag = 20*log10([0.473 0.343 0.144 0.052 0.010])';
phase = -[19.2 45.2 70.8 79.4 88.7]';

[MAG,PHASE,W] = bode(sys,{10,30000});

figure()
subplot(2,1,1)
hold on
semilogx(f,mag,'ro','LineWidth',1.5);
semilogx(W(:),20*log10(MAG(:)),'LineWidth',1.5);
set(gca,'xscale','log')
xlim([0,30000])
xticklabels({})
ylabel('Magnitude (dB)');
```

```
subplot(2,1,2)
hold on
semilogx(f,phase,'ro','LineWidth',1.5);
semilogx(W(:),PHASE(:),'LineWidth',1.5);
set(gca,'xscale','log')
axis([0, 30000, -90, 0]);
xlabel('Frequency (Rad/s)');
ylabel('Phase (\circ)');
```