# Feedback Control with an RRC Circuit

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**Abstract**

This report describes the effects of feedback control with a proportional gain controller on an RRC circuit. Analysis is based on performance measures such as rise-time and steady-state error. Robustness of the system is analyzed with a Nyquist plot, and the stability at varying proportional gain is represented through root-locus.

## 1 Introduction

The RRC circuit is an easy to measure, small, cheap, and overall great representation of a first order system. Figure 1 shows the schematic of the circuit and the terminals where the voltage is supplied and measured. The RRC circuit used in this lab consists of an $R_1 = 5040\Omega$, $R_2 = 4976\Omega$, and a capacitor value, experimentally determined from the previous lab, to be $C_1 = 2.22\mu\text{F}$. The transfer function for the RRC circuit is numerically simplified from with equation 1 and the adjusted model is shown in Figure 2.

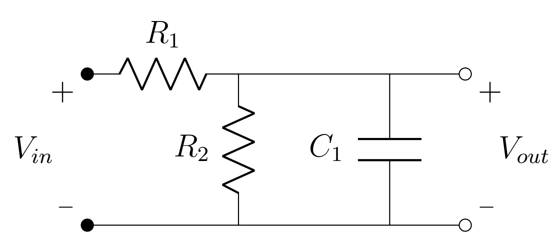



Figure 1: RRC Circuit - a simple first order system representation

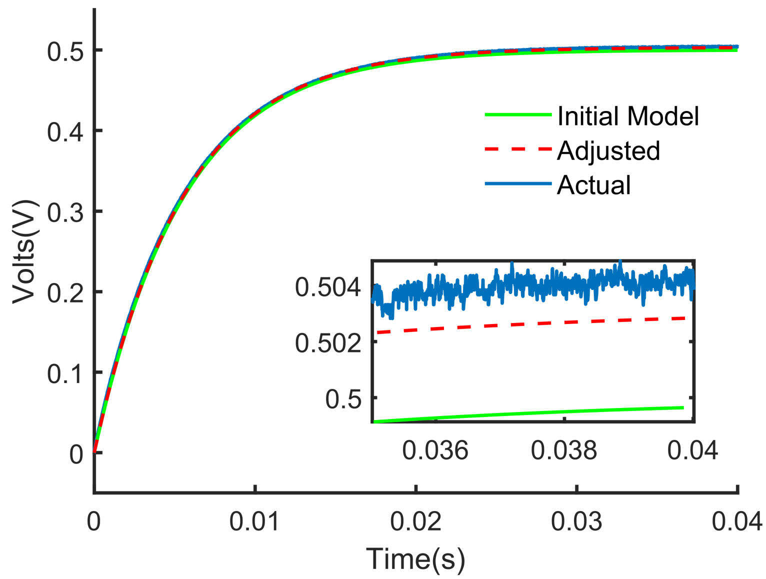



Figure 2: The RRC step response adjusted with the values of $R_1$ and $R_2$, fitted with a capacitor value of 2.22 µF

$$G(s) = \hat{k}\frac{\sigma}{s+\sigma} = 0.502\frac{179.84}{s+179.84}, \tag{1}$$

The performance characteristics of this circuit correspond to a steady state error of 0.498 V and an experimental rise time of 12.233 ms, verified with Equation 2.

$$t_r = \frac{2.2}{\sigma} = \frac{2.2}{179.84} = 12.233(10)^{-3} \tag{2}$$

This is the plant that will be used with feedback control in this lab and for all theoretical calculations. The block diagram for a proportional gain controller in negative feedback is found in Figure 3.

## 1.1 Experimental Setup

The setup for this lab removed the function generator, and instead we are using the NI myRIO to produce signals for the circuit. This hardware is able to use LabVIEW to create the voltages needed for analysis of the system (analog out). In addition, the analog inputs of the myRIO are used to sample

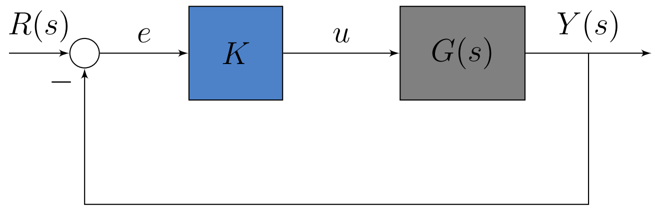



Figure 3: Closed Loop Block Diagram for Proportional Gain

the output waveform from the RRC circuit to implement a proportional controller. The screw terminal blocks on the right side of the device were configured in LabVIEW to output the 1V step (square wave for multiple sets of data). The oscilloscope is still being used to obtain a better resolution of the voltage output. See Figure 4 for a more detailed view of the setup and wiring. The red wire corresponds to the input of the plant, white is the output, and the other wires (blue and green) are used for grounding and the negative terminal of the analog input.

# 2 Performance

Due to the RRC circuit being a first order system, the performance metrics used to quantify the quality of the system are going to be rise time and steady state error. Without feedback, the system is open loop and has a DC-gain of approximately 0.502 and the rise time was clocked in at 12.233 ms as seen in Figure 2. In order to increase the performance of the system, the adjusted model of the RRC circuit will be used in closed loop control.

## 2.1 Closed Loop and Proportional Gain

One of the simplest controllers to implement in closed loop is a proportional gain controller, where the error (difference between reference and current value) is amplified by a constant $K$ and applied to the plant. The closed loop transfer function is described in Equation 3; see Figure 3 for the block diagram.

$$T(s) = \frac{kG(s)}{1 + kG(s)} = \frac{90.28K}{s + 90.28K + 179.84} \tag{3}$$

The main consideration to watch when using proportional gain is to ensure that the controller does not saturate, by attempting to output a value greater

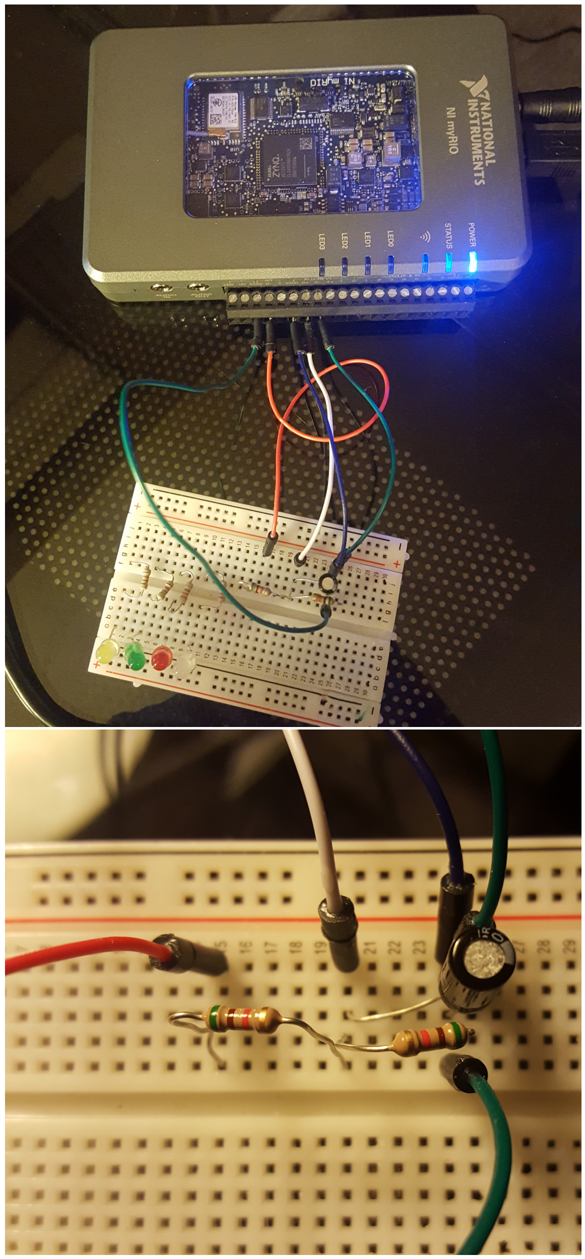



Figure 4: Experimental Setup of RRC with input signal $u$ (Red), Output signal (White), and ground (Greens and Blue)

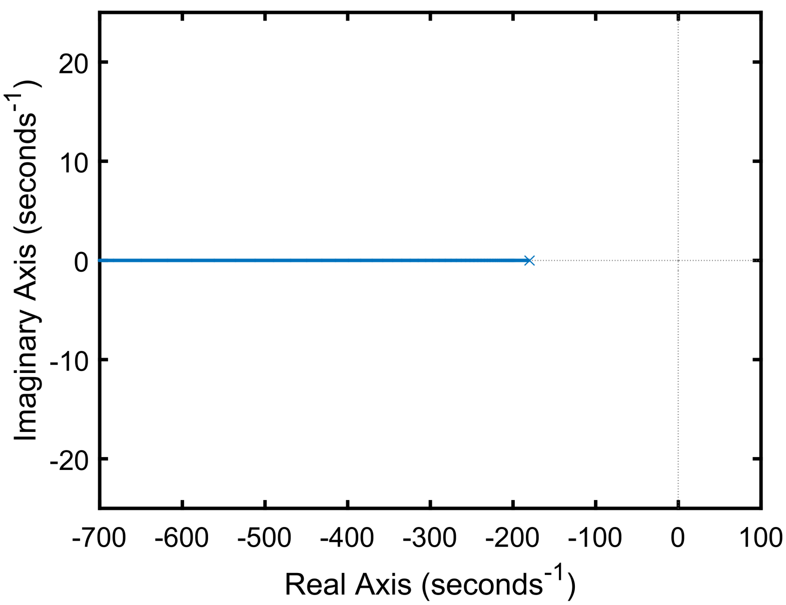



Figure 5: Root Locus plot of RRC circuit showing how the first order system responds to proportional gain

than the voltage rails of the controller. If this occurs, then the system is no longer a liner time-invariant system and the methods used to classify it no longer apply. For a first order system, such as the RRC circuit, closed loop feedback is used to enhance performance and shift the pole of system left (resulting in a shorter rise-time). The myRIO is capable of a producing signals of magnitude $\pm$ 10V, so if the output voltage from the proportional controller exceeds that it will become saturated.

Additionally a greater gain results in a lower steady state error, approaching zero as gain approaches infinity. This can be visualized in a root-locus plot, which shows the potential of improvement with varying proportional gains, shown in Figure 5. As seen in the figure, the circuit remains stable for all gain because there are no poles on the right-hand plane. By increasing the proportional constant $K$, the only pole is shifted further and further left. To implement the ideas above, the myRIO controller was set to sample the output of the RRC circuit in intervals of 1 ms in proportional gain. The step response for varying gains can be found in Figure 6, alongside the response of

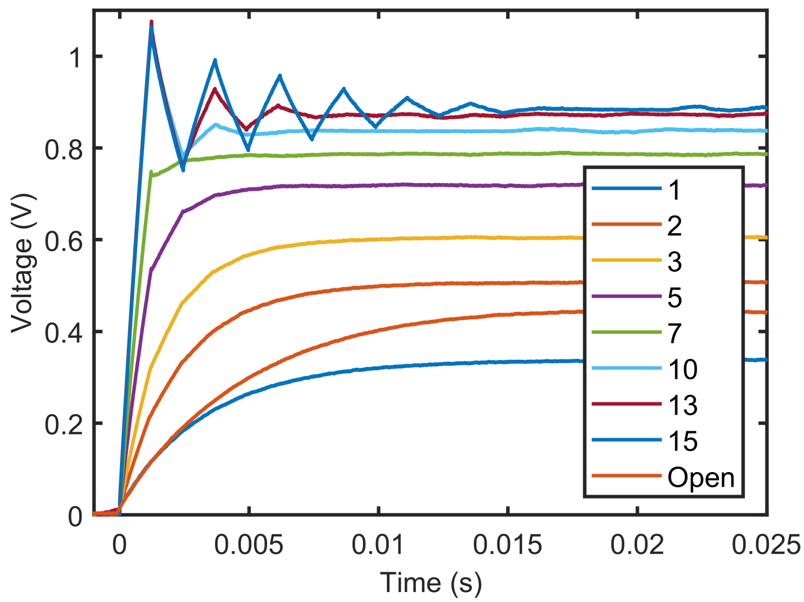



Figure 6: Step response of the RRC circuit with a proportional gain controller for varying K values to a reference 1V step.

the open loop system. The rise time decreases as the proportional constant K increases, however there are oscillations that occur once $K = 10$ due to the sampling rate of the myRIO. At the best non oscillatory gain ($K = 7$), the rise time of the circuit is approximately 2.2ms with a DC-gain of 0.788.

# 3 Sensitivity Function

The sensitivity function is useful in controls because of its insight into system robustness. Robustness is a measurement determined by how ”close” the system is to becoming unstable. Because the RRC circuit is a first order system, placing it in proportional gain feedback will still result in a stable system (with a model of reasonable certainty). However, by looking at the sensitivity function we are able to find the maximum value and relate it to robustness on a Nyquist plot.

## 3.1 Measuring Robustness

In order to get to the Nyquist plot, there are a couple of definitions and derivations that need to be laid out. The loop transfer function for negative feedback $L(s) = KG(s)$ which is equal to the controller and plant put into one block diagram. Looking at that version of the closed loop transfer function, also known as the complementary sensitivity function ($T$), the sensitivity function ($S$) can be derived. By definition $S + T = 1$, so the sensitivity function can be defined in Equation 4. For the RRC circuit, the maximum of the sensitivity function (nominal sensitivity peak $M_s$) will provide insight into the distance closest to the point -1 on the Nyquist plot. In fact, it is the inverse of this value that is the distance. This is important because the closer the plot is to -1, the more likely it is that model uncertainty could cause the plot to encircle -1 and therefore become an unstable system. This measurement $1/M_s$ is a measure of robustness. For our first order system, looking at the bode plot of the sensitivity function (Figure 7) shows that the nominal sensitivity peak $M_s = 1$, this means that at any gain the system will remain stable will not encircle the point -1 and that the gain margin is infinity, and the phase margin is 90 degrees. The sensitivity function provides some insight into the robustness of the system, but there is another way of obtaining the gain and phase margins. To verify this from another angle, both metrics can be obtained from the frequency response of the closed loop system. Looking at the bode plot of the closed loop transfer function, found in Figure 7, note that the function does not cross the 0dB mark on the magnitude plot meaning that you can increase the gain infinitely and it still will not cross 0dB. Likewise, the phase plot does not cross -180 degrees meaning that the closest the system will be to -180 degrees phase is when it is at -90 degrees. The result: infinite gain margin and 90 degree phase margin.

$$S(s) = \frac{1}{1 + L(s)} = \frac{s + 179.84}{s + 90.28K + 179.84} \tag{4}$$

# 4 Nyquist

Relating what was previously stated to a Nyquist plot, the phase margin is how far you can rotate the shape before it encircles -1, and gain margin is how much you can amplify the shape before reaching -1. See Figure 8 for more information.

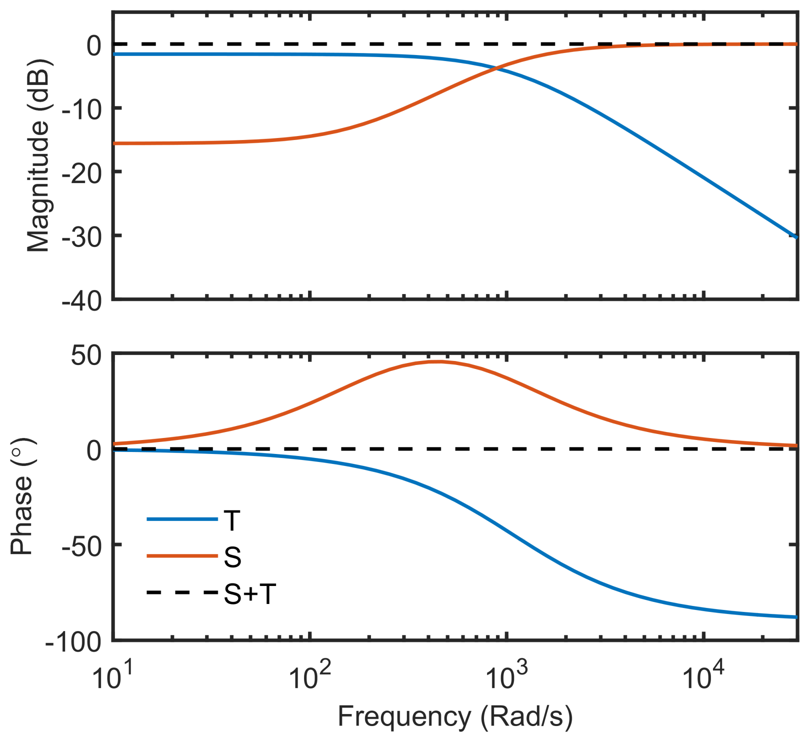



Figure 7: Bode Plot for the sensitivity and complementary sensitivity function with the Bode plot of their sum to show that the RRC circuit has a Gain Margin of infinity and a Phase Margin of 90 degrees

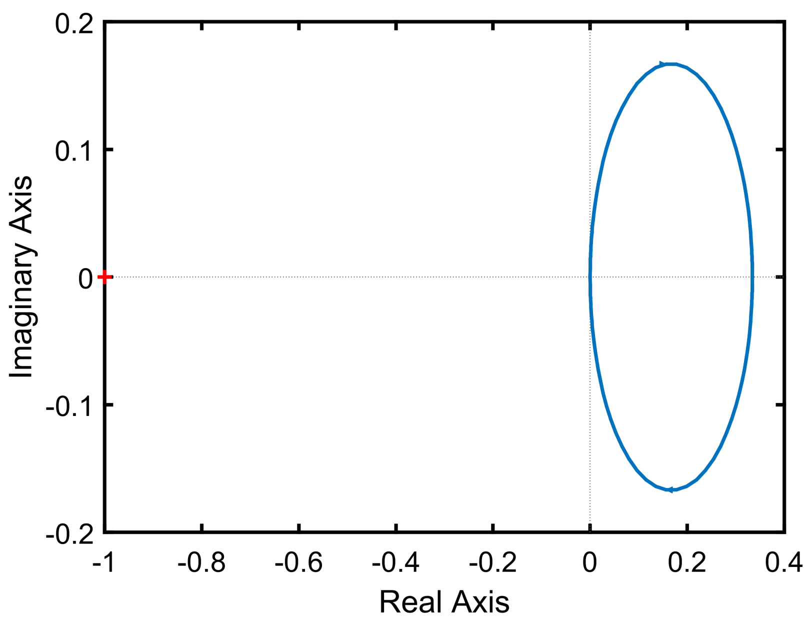



Figure 8: Nyquist Plot for the RRC circuit (1st order system) with a Gain Margin of infinity and a Phase Margin of 90 degrees

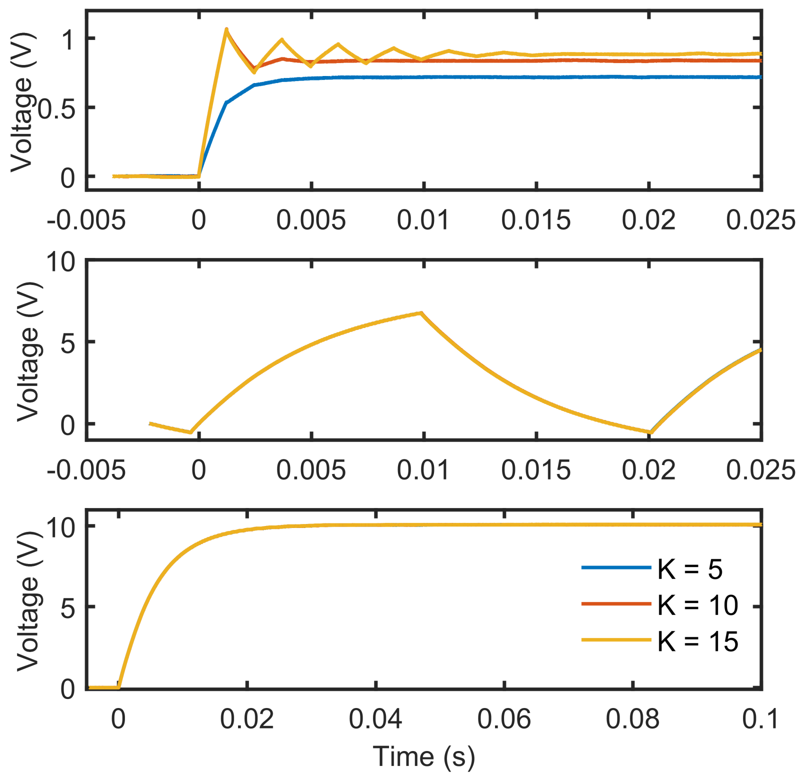



Figure 9: Step responses of the system to a 1V reference with proportional gain and varying sampling rates. Top: 1 ms Sampling rate, Middle: 10ms Sampling rate, Bottom: 100ms Sampling rate.

# 5 Sampling Rate

The sampling of the controller (the myRIO) has a large effect on the outcome of the appearance of the RRC circuit output. For example, any sampling rate below the risetime of the system will miss the curve entirely, especially at high gain. Figure 9 shows the outcomes of several configurations of gain and sampling rate. It is worth noting that for the lower sampling rates and high gain, the voltage output from the circuit converges to a single waveform.

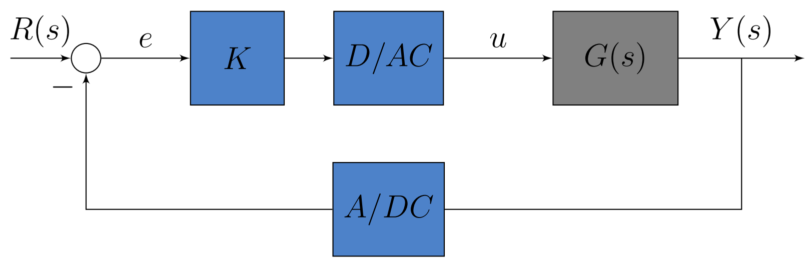



Figure 10: Closed Loop Block Diagram of signal conversion between myRIO(Blue) and input/output of circuit (Gray) using an digital to analog converter(D/AC) and an analog to digital converter (A/DC).

## 5.1 Fourier Transform and the Comb Function

It is not surprising to see that the continuous theoretical representation of a proportional controller does not align exactly with the implementation in a discrete setting. There are a combination of factors at play, but the majority lie within the transformation of the signal from digital to analog and how the sampling rate affects the Fourier Transform of the signal. The first point is that the myRIO is acting as a digital controller and changing the digital signal to analog, and visa versa when receiving the output signal from the RRC circuit. This is illustrated in Figure 10. The blocks in blue are within the myRIO, and the plant is the RRC circuit. This is more of a minor point for the purposes of this lab, the more influential is the sampling frequency. For one, discrete sampling in the time domain results in periodicity for the frequency domain. This can be shown using the Dirac delta function at intervals of $T_0$ (also known as the comb function) and is instrumental in determining the frequency representation of a discrete signal. The sampling property of the delta function allows the comb function to sample at a continuous waveform at a discrete interval. Taking the Fourier Transform of this function results in a periodic version of the continuous signal's Fourier Transform. However, as the sampling rate goes to infinity the periodicity in the Fourier Transform decreases until it becomes the continuous signal's Transform. For the case in this lab, the sampling rate varied and resulted in the convergence of the output signal possibly due to the the reasons above, however it also may be simply due to the fact that the sampling rate was not at a great enough resolution to capture the movement during the rise time of the system. The controller could then be making error determinations from "low" and "high" rather than the gradual increase during the circuit excitation.

# 6 Conclusion

Implementing a proportional controller with the myRIO, the RRC circuit was able to respond with a rise time five times as fast as the open loop configuration and with a DC-gain of 0.778 at a stable gain of $K = 7$. Higher gain resulted in a larger DC-gain, but ultimately oscillated due to the low sampling frequency of 1 kHz. Understanding the digital performance of the myRIO and the limitations of a digital controller will provide more insight into the reason for a oscillatory 1st order system at high gain. The stability of the system is guaranteed if the model is accurate and without disturbance due to the nature of 1st order systems. The RRC circuit is also incredibly robust with a gain margin of infinity and a phase margin of 90 degrees. The inverse of the nominal sensitivity peak is also 1, guaranteeing that the system will not encircle the point -1 on a Nyquist plot. Moving forward, a more complex controller and a higher sampling rate could further increase the performance of the system, allow for a higher gain feedback loop, and possible implement disturbance rejection.

# A Code

## A.1 Creating Plots and Measuring Performance

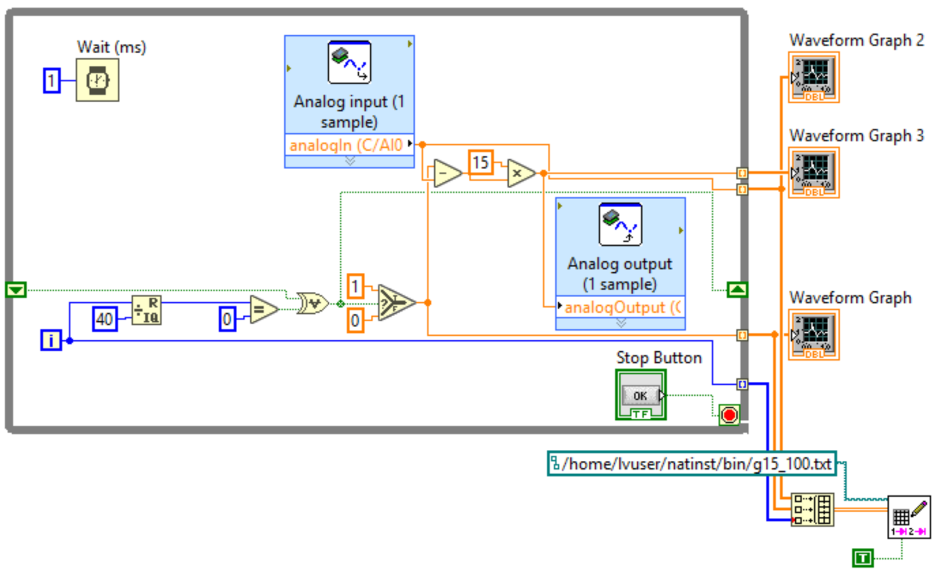
```
%% Lab02 - scope Data
% Riley Kenyon
% 2/25/2019
% Labview const = 0.1ms, period of 40ms
%---------------------------------------------------------------------------
close all;

% Main Figure
figure();
hold on
names = [1 2 3 4 5 7 10 13 15];
samples = 1;
```

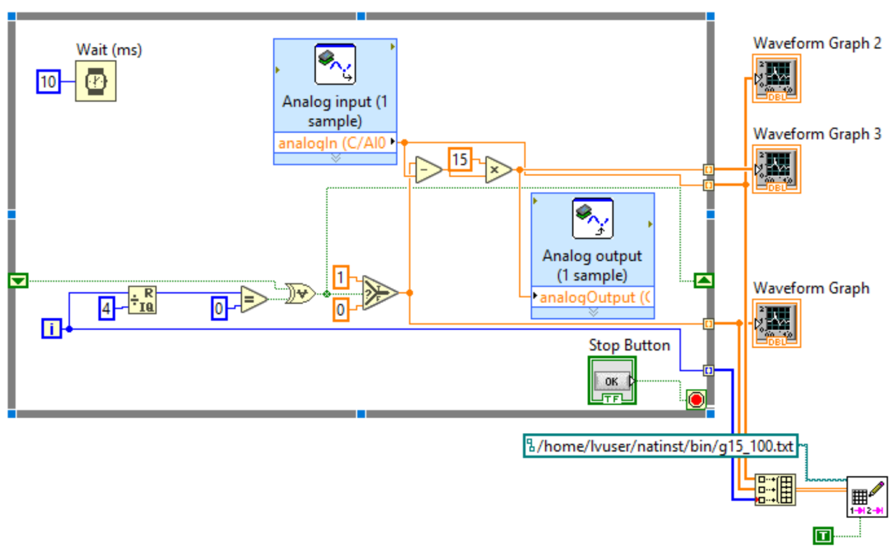
```
for i = 1:length(names)
    fname = sprintf('g%02d_%03d.csv',names(i),samples);
    %disp(fname);
        if (exist(fname,'file') ~= 0)
            data = csvread(fname,7,0);
            index = 3;
            data(:,2) = data(:,2) - data(index,2);
            data(:,1) = data(:,1) - data(index,1);
            index = find(data(:,2)>0.015,1);
            data(:,1) = data(:,1) - data(index,1);
            plot(data(:,1),data(:,2),'LineWidth',1.5)
        end
        fprintf("Rise Time K = %01d : %0.6f \n",names(i), risetime(data(:,2),3.333
end
data = csvread('o1.csv',7,0);
index = 3;
data(:,2) = data(:,2) - data(index,2);
data(:,1) = data(:,1) - data(index,1);
index = find(data(:,2)>0.015,1);
data(:,1) = data(:,1) - data(index,1);
fprintf("Rise Time OpenLoop : %0.6f \n", risetime(data(:,2),length(data)/(data(end
plot(data(:,1),data(:,2),'LineWidth',1.5)
axis([-0.001,0.025,0,1.1]);
legend('1','2','3','5','7','10','13','15','Open')

% Figure with different sampling rates
% Acquire Data
clear names samples;
names = [5 10 15];
samples = [001 010 100];
figure()
for i = 1:length(samples)
    subplot(3,1,i)
    %xlim([0,0.025])
    hold on
    for j = 1:length(names)
    fname = sprintf('g%02d_%03d.csv',names(j),samples(i));
        if (exist(fname,'file') ~= 0)
```

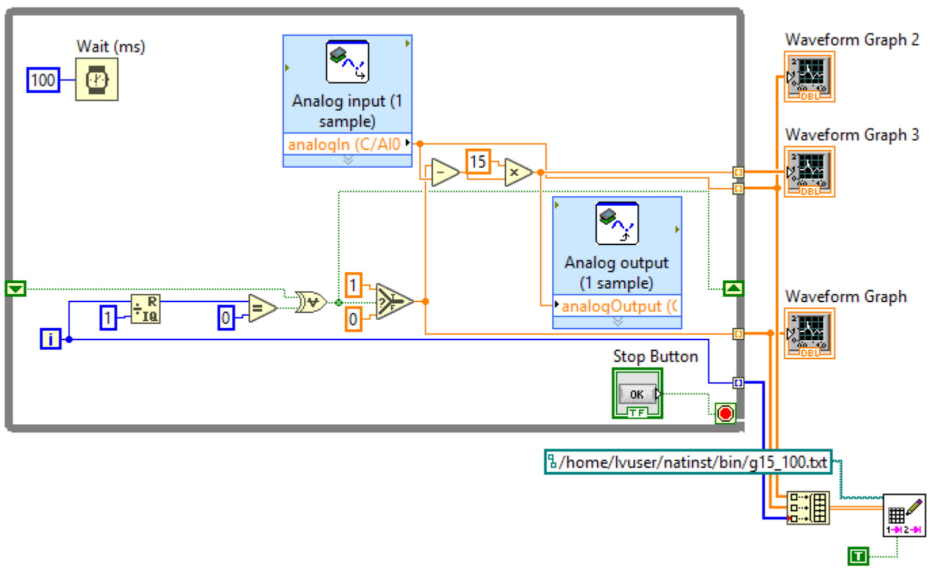
```
            data = csvread(fname,7,0);
            index = 3;
            data(:,2) = data(:,2) - data(index,2);
            data(:,1) = data(:,1) - data(index,1);
            index = find(data(:,2)>0.015,1);
            plot(data(:,1) - data(index,1),data(:,2),'LineWidth',1.5)
        end
    end
end
subplot(3,1,1)
ylim([-0.1 1.2]);
xlim([-0.005,0.025])
subplot(3,1,2)
ylim([-1,10])
xlim([-0.005,0.025])
subplot(3,1,3)
ylim([-0.1,11])
xlim([-0.005,0.1])
legend('K = 5','K = 10','K = 15')

% More Plots
%--------------------------------------------------------------------
clear all;
G = tf(90.27968,[1,179.84]);
k = 10;
L = k*G;
T = minreal(L/(1+L));
S = minreal(1/(1+L));
figure()
rlocusplot(G);
%axis([-4 4 -4 4]);
pos = get(gcf, 'Position');
set(gcf, 'Position',[pos([1 2 4]) pos(4)]);
%title = get(gca,'Title');
H = feedback(G,1);
figure()
w = linspace(-20*pi,20*pi,10000);
[RE,IM,wout] = nyquist(H);
```

```
RE = squeeze(RE);
IM = squeeze(IM);
%plot(RE,IM,RE,-IM);
nyquist(H);
% figure()
% margin(H);

% BODE PLOTS
%--------------------------------------------
[MAG,PHASE,W] = bode(T,{10,30000});
[MAGS,PHASES,WS] = bode(S,W);
[MAGST,PHASEST] = bode(S+T,W);
figure()
subplot(2,1,1)
hold on
semilogx(W(:),20*log10(MAG(:)),'LineWidth',1.5);
semilogx(WS(:),20*log10(MAGS(:)),'LineWidth',1.5);
semilogx(WS(:),20*log10(MAGST(:)),'k--','LineWidth',1.5);
set(gca,'xscale','log')
xlim([0,30000])
ylim([-40,5])
xticklabels({})
ylabel('Magnitude (dB)');

subplot(2,1,2)
hold on
semilogx(W(:),PHASE(:),'LineWidth',1.5);
semilogx(WS(:),PHASES(:),'LineWidth',1.5);
semilogx(WS(:),PHASEST(:),'k--','LineWidth',1.5);
set(gca,'xscale','log')
xlim([0 30000]);
xlabel('Frequency (Rad/s)');
ylabel('Phase (\circ)');
legend('T','S','S+T')
```

## A.2 Labview


Wait (ms)
Analog input (1 sample)
analogIn (C/AI0
Analog output (1 sample)
analogOutput (C
Stop Button
Waveform Graph 2
Waveform Graph 3
Waveform Graph
/home/lvuser/natinst/bin/g15_100.txt


Wait (ms)
Analog input (1 sample)
analogIn (C/AI0
Analog output (1 sample)
analogOutput (C
Stop Button
Waveform Graph 2
Waveform Graph 3
Waveform Graph
/home/lvuser/natinst/bin/g15_100.txt



Wait (ms)
100
Analog input (1 sample)
analogIn (C/AI0
15
Analog output (1 sample)
analogOutput (C
1
0
Waveform Graph 2
Waveform Graph 3
Waveform Graph
Stop Button
/home/lvuser/natinst/bin/g15_100.txt